Standard Errors

Linear and Generalized Linear Models

Math 398

Advance Research Investigations

Bayesian Approach

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Learning Objectives

- Standard Errors
  - Linear Regression
  - GLM
- Sampling Distributions

Linear Regression

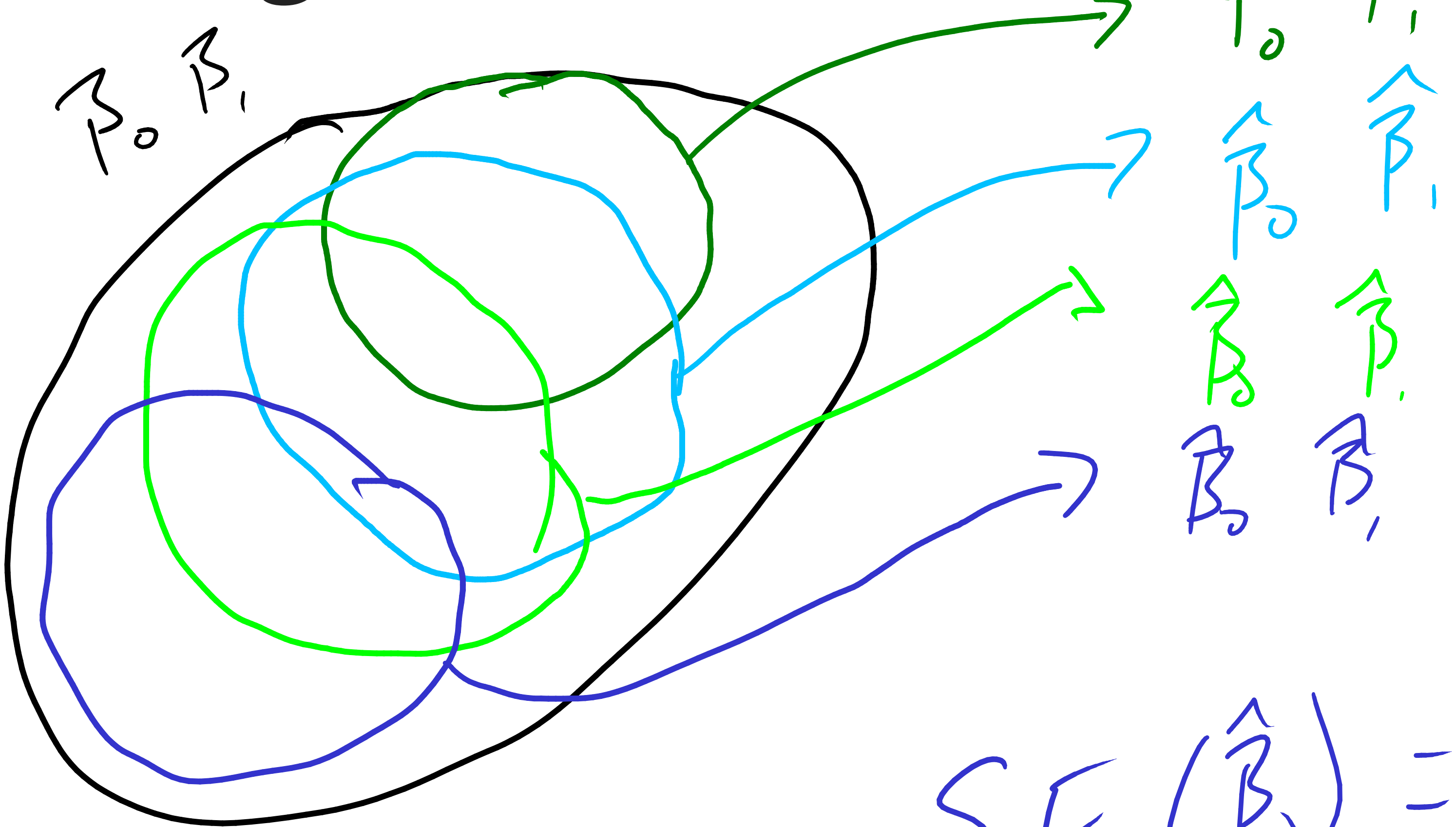
Standard Errors

- Find the variance of the estimate
- Find the information matrix
- Use for Inference

$\hat{\beta}_0 \quad \hat{\beta}_1$

$Y = \hat{\beta}_0 + \hat{\beta}_1 X$

Finding the Variance



$SE(\hat{\beta}_0)$ = Variation of $\hat{\beta}_0$ due to random sampling

$\hat{\beta}_0 \pm MOE$

$MoE = SE \cdot CV$

critical value

$SE(\hat{\beta}_0) = \sqrt{Var(\hat{\beta}_0)}$ $\qquad$ $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$ $\qquad$ $\bar{Y} \sim N(\beta_0 + \beta_1 \bar{x}, \frac{\sigma^2}{n})$

$Var(\hat{\beta}_0)$ $\qquad$ $Y_i \sim N(\beta_0 + \beta_1 X_i, \sigma^2)$

$$Var(\bar{Y} - \hat{\beta}_1 \bar{X}) = \underbrace{Var(\bar{Y})}_{\frac{\sigma^2}{n}} + \bar{X}^2 Var(\hat{\beta}_1)$$

$$\hat{\beta}_1 = \Sigma \frac{(x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2}$$

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 f(y)\, dy$$

Estimate for σ^2

$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}(Y_i - \boldsymbol{X}_i^{\mathrm{T}}\hat{\boldsymbol{\beta}})^2$$

$$X_i^T \hat{\beta} = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

Standard Errors of β's

wing span = body mass
+
Beak size
+
species
3
seagull duck condor

$$SE(\hat{\beta}_0) = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 \hat{\sigma}^2}{n \sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

$$SE(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

$bM = \beta_0 + \beta_1 WS + \beta_2 BS + \beta_3 \text{duck} + \beta_4 \text{condor}$

Standard Errors Matrix Form

$$Var(\hat{\boldsymbol{\beta}}) = (\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X})^{-1}\hat{\sigma}^2$$

$p = \#\text{ predictors} + 1$

$p \times p$

5

5×5

$$X = \begin{array}{ccccc} \beta_0 & \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ & ws & bs & Do & con \end{array} \left(\begin{array}{ccccc} \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \right)$$

$$5 \quad \begin{pmatrix} Var(\beta_0) & & & Cov(\beta_i, \beta_j) \\ & Var(\beta_1) & & \\ & & \ddots & \\ Cov(\beta_i, \beta_j) & & & Var(\beta_4) \end{pmatrix}$$

Generalized Linear Models

Large Sample Theory

Let $X_1, \ldots, X_n$ be a random sample from a distribution with parameter θ. Let $\hat{\theta}$ be the MLE estimator for a parameter θ. As $n \to \infty$, then $\hat{\theta}$ has a normal distribution with mean θ and variance $1/nI(\theta)$, where

[Math Processing Error]

$$SE(\beta_0) = \sqrt{\frac{1}{nI(\beta_0)}}$$

$$I(\beta_0) = -E\left[\frac{d^2 \ell(\beta)}{d\beta_0^2}\right]$$

$$\hat{\beta}_0 \sim N\left(\beta_0, \frac{1}{nI(\beta_0)}\right)$$

Sampling Distributions

Sampling Distributions

ϕ known

Poisson Binomial

$\phi = 1$

$$\frac{\hat{\beta}_j - \beta_j}{\mathrm{se}(\hat{\beta}_j)} \sim N(0, 1)$$

$\hat{\beta}_j \sim N(\beta_j, Var(\hat{\beta}_j))$

ϕ unknown

Normal

$\phi = \sigma^2$

$$\frac{\hat{\beta}_j - \beta_j}{\mathrm{se}(\hat{\beta}_j)} \sim t_{n-p'}$$

$df = n - p'$

$p' = \#$ predictors $+1$

$t_{DF} \rightarrow Z \sim N(0,1)$

$DF \rightarrow \infty$